Baryon form factors in a diquark–quark bound state description[†]

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Abstract

Nucleon form factors are calculated in a relativistic diquark–quark picture based on the Nambu–Jona-Lasinio model. The nucleon wave function is obtained in a static approximation to the quark exchange interaction between the valence quark and the diquark. We evaluate the valence quark and 0^+–diquark contribution to the nucleon electromagnetic and weak currents. We find reasonable electric charge radii, magnetic moments as in the additive diquark model, and $g_A \approx 1$. We discuss the dependence on the model parameters.

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A quantitative understanding of baryon form factors at low energies in terms of quark degrees of freedom remains a challenging problem. Attempts to base a low–energy description of baryons on QCD usually rely on the limit $N_C \to \infty$, in which a soliton picture emerges. While properly describing the effects of the non–perturbative meson cloud, this approach suffers from difficulties related to the breaking of translational invariance. For many purposes, such as *e.g.* the calculation of timelike nucleon form factors, it is important to maintain proper relativistic kinematics. A description of the nucleon as a relativistic bound state can be realized in quark models with an effective 2–body interaction [1, 2, 3]. For $N_C = 3$, this naturally leads to the concept of diquarks as effective constituents [4]. The simplest such model is a Nambu–Jona-Lasinio (NJL–) model with a local (current–current) interaction in the $\bar{3}$–color channel. A corresponding color–singlet interaction describes the spontaneous breaking of chiral symmetry and provides a reasonable account of meson properties [5, 6]. The NJL model can be rewritten as an effective theory of mesons and baryons [2], in which bound states of composite diquarks and quarks occur due to quark exchange. The spectrum of $\frac{1}{2}^+$–baryons has been calculated using a static approximation for the quark exchange interaction [7]. The Faddeev equation for the nucleon has also been solved exactly [8], including also the 1^+–diquark channel [9, 10].

Here, we apply this description to the study of the nucleon form factors at low momentum. We consider the simplest case of a bound state of a 0^+–diquark and a valence quark and take the baryon wave functions in the static approximation to the quark exchange. Given the complexity of any relativistic description of baryons, it is useful to first explore this simple approximation. The present study extends a previous analysis of the form factors in an additive diquark–quark scheme [11]. Contrary to that approach, we now derive the relativistic nucleon wave function within the model quark dynamics in a crude but consistent approximation. Our intention is to estimate the role of diquark and valence quark currents in the nucleon electromagnetic and weak form factors. Moreover, we wish to point out some general features of the bound state description of baryons, which are likely to persist in more elaborate calculations with exact Faddeev wave functions.

The basis of our approach is the effective baryon action, which is obtained by rewriting the partition function of the NJL–model in terms of composite diquark and baryon fields [2, 3]. It is defined in terms of the baryon Greens function,

$$G_B = [G_0^{-1} - H]^{-1} + \Delta G_B. \tag{1}$$

Here,

$$G_0(k, k'; P) = \mathcal{D}(\tfrac{1}{2}P - k)G(\tfrac{1}{2}P + k')\delta(k - k') \tag{2}$$

is the product of the scalar diquark propagator, $\mathcal{D}$, which is the solution of the diquark Bethe–Salpeter equation of the NJL model [2], and the quark propagator, $G(k) = (\not{k} - $

$M)^{-1}$. We consider the isospin limit, $M_u = M_d \equiv M$. P and k are the total and relative four–momentum of the system. Furthermore,

$$H(k, k') \quad = \quad \Gamma G(-k - k')\Gamma \tag{3}$$

is the quark exchange interaction. For a scalar diquark, $\Gamma = i\gamma_5 C\epsilon$, where C is the charge conjugation matrix and ϵ the generator of the $\bar{3}$–representation of the color group. The contact term in eq.(1), ΔG_B, will not be important in the following. Baryon masses and bound state wave functions are obtained as color–singlet solutions of the Faddeev–type equation

$$\int \frac{d^4 k'}{(2\pi)^4} G_B^{-1}(k, k'; P)\psi(k'|P) \quad = \quad 0, \qquad P^2 = m_B^2. \tag{4}$$

In the static approximation for the quark exchange [7],

$$H(k, k') \quad \to \quad -\Gamma \frac{1}{M}\Gamma, \tag{5}$$

the baryon wave function in eq.(4) can be written as

$$\psi(k|P) \quad = \quad \mathcal{D}(\tfrac{1}{2}P - k)G(\tfrac{1}{2}P + k)\phi(P). \tag{6}$$

The baryon amplitude, $\phi(P)$, is then obtained as the solution of a Dirac–type equation,

$$\left(A(P^2)\slashed{P} + B(P^2) - M\right)\phi(P) \quad = \quad 0, \tag{7}$$

where the effective Dirac operator is defined as

$$A(P^2)\slashed{P} + B(P^2) \quad = \quad \int_{\Lambda_B} \frac{d^4 k}{(2\pi)^4} \mathcal{D}(\tfrac{1}{2}P - k)G(\tfrac{1}{2}P + k). \tag{8}$$

In this approximation the baryon amplitude does not depend on the relative momentum of the diquark–quark pair. The integral in eq.(8) is defined with a momentum cutoff, Λ_B, which effectively controls the width of the baryon wave function. (Λ_B is not identical to the NJL cutoff, *cf.* below.) We normalize the on–shell baryon amplitude by the condition

$$\bar{\phi}(P) \left[P^\mu \frac{\partial}{\partial P_\mu} \left(A(P^2)\slashed{P} + B(P^2) \right) \right]_{P^2 = m_B^2} \phi(P) \quad = \quad m_B. \tag{9}$$

Eq.(9) fixes the correct relativistic dispersion law for the c.o.m. motion of the bound state and is identical to the usual normalization condition for Bethe–Salpeter amplitudes.

Baryon form factors are calculated by coupling to the free baryon action an electromagnetic or weak gauge field, $\mathcal{A}_\mu$, by minimal substitution at quark level. One then obtains the current of the baryon field as the corresponding functional derivative of the

inverse baryon propagator, eq.(1). The matrix element of the baryon electromagnetic current for a transition $P_\mathrm{i} \to P_\mathrm{f}$, where $P_\mathrm{i}^2 = P_\mathrm{f}^2 = m_B^2$, is in general given by

$$J_B^\mu(P_\mathrm{f}, P_\mathrm{i}) = \int \frac{d^4k}{(2\pi)^4} \int \frac{d^4k'}{(2\pi)^4} \, \bar\psi(k|P_\mathrm{f}) \frac{\delta G_B^{-1}}{\delta \mathcal{A}^\mu}(k, k'; P_\mathrm{f}, P_\mathrm{i}) \, \psi(k'|P_\mathrm{i}) \tag{10}$$

$$= \left(F_B^\mathrm{e}(Q^2) - F_B^\mathrm{m}(Q^2) \right) 2m_B \frac{(P_\mathrm{f} + P_\mathrm{i})^\mu}{(P_\mathrm{f} + P_\mathrm{i})^2} + F_B^\mathrm{m}(Q^2)\gamma^\mu. \tag{11}$$

Here, $Q = P_\mathrm{f} - P_\mathrm{i}$, and $F_B^\mathrm{e}, F_B^\mathrm{m}$ are the baryon electric and magnetic form factor. The baryon current consists of terms describing the coupling of the external field to the diquark propagator, the quark propagator and the quark exchange. (The contact term in eq.(1) does not contribute to the baryon current on the mass shell.) In the static approximation, eq.(5), it is consistent to neglect the quark exchange current. Note that the quark exchange also does not contribute to the normalization of the baryon amplitude, *cf.* eq.(9). Thus, in the static approximation, eq.(10) becomes

$$J_B^\mu(P_\mathrm{f}, P_\mathrm{i}) = \bar\phi(P_\mathrm{f}) \left(\Gamma_{B,\mathrm{diq}}^\mu(P_\mathrm{f}, P_\mathrm{i}) + \Gamma_{B,\mathrm{quark}}^\mu(P_\mathrm{f}, P_\mathrm{i}) + C(\Lambda_B)\gamma^\mu \right) \phi(P_\mathrm{i}) \tag{12}$$

$$\Gamma_{B,\mathrm{diq}}^\mu(P_\mathrm{f}, P_\mathrm{i}) = \int_{\Lambda_B} \frac{d^4k}{(2\pi)^4} \mathcal{D}(\tfrac{1}{2}P_\mathrm{f} - k)\mathcal{F}^\mu(\tfrac{1}{2}P_\mathrm{f} - k, \tfrac{1}{2}P_\mathrm{i} - k)\mathcal{D}(\tfrac{1}{2}P_\mathrm{i} - k)G(\tfrac{1}{4}P_\mathrm{f} + \tfrac{1}{4}P_\mathrm{i} + k)$$

$$\tag{13}$$

$$\Gamma_{B,\mathrm{quark}}^\mu(P_\mathrm{f}, P_\mathrm{i}) = \int_{\Lambda_B} \frac{d^4k}{(2\pi)^4} \mathcal{D}(\tfrac{1}{4}P_\mathrm{f} + \tfrac{1}{4}P_\mathrm{i} - k)G(\tfrac{1}{2}P_\mathrm{f} + k)\Gamma_q^\mu G(\tfrac{1}{2}P_\mathrm{i} + k) \tag{14}$$

We note that the on–shell baryon current defined by eqs.(13, 14) is transverse. Here,

$$\mathcal{F}^\mu(k, k') = (Q_u + Q_d)Z_{0^+}^{-1} \left\{ (k + k')^\mu F_{0^+}(q^2, k^2, k'^2) + q^\mu G_{0^+}(q^2, k^2, k'^2) \right\}, \tag{15}$$

with $q = k - k'$, is the off–shell e.m. vertex function of the extended 0^+–ud–diquark [11, 12]. The valence quark is taken as pointlike, $\Gamma_q^\mu = iQ_{u,d}\gamma^\mu$, corresponding, respectively, to a proton or a neutron. ($Q_{u,d} = \frac{2}{3}, -\frac{1}{3}$ are the quark charges.) In the approach based on [2], a valence quark form factor would arise if dynamical meson exchange were taken into account[1]. Furthermore, we have included in eq.(12) a counter term, $C(\Lambda_B)\gamma^\mu$, which we choose to fix charge conservation at $Q^2 = 0$. This is necessary when working with a momentum–space cutoff. However, in the numerical calculations we find that the violation of charge conservation is small, *i.e.*, $C(\Lambda_B) \ll 1$. Note also that the counter term does not affect the charge radii.

The composite diquark propagator and the corresponding e.m. vertex function, eq.(15) have been determined within the gauge invariant proper–time regularized NJL model

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[1] A consistent treatment of meson exchange effects would require the inclusion of meson exchange in the quark–quark interaction and the diquark–quark binding

[11, 12]. To facilitate the evaluation of the loop integrals in eqs.(13, 14), we approximate the full diquark propagator by its pole form,

$$\mathcal{D}(k) \;\;\rightarrow\;\; \frac{Z_{0^+}}{k^2 - m_{0^+}^2}, \tag{16}$$

where the pole and the residue are determined from the exact solution. This is justified if the cutoff defining the baryon wave function, Λ_B, is not too large, which is the case in the calculations below. Consistently with the pole form of the propagator, eq.(16), we make an on–shell approximation[2] for the composite diquark vertex function, *i.e.*, we replace in eq.(15) the diquark form factor $F_{0^+}(q^2, k^2, k'^2)$ by its value at $k^2 = k'^2 = m_{0^+}^2$. (The longitudinal part of eq.(15) does not contribute to the on–shell baryon form factor.) For regularization of the diquark–quark loop integrals in eqs.(13, 14) we employ a sharp euclidean cutoff, Λ_B, after introducing Feynman parameters, *cf.* [6]. (We have also done calculations with a proper–time–type regularization for the diquark–quark loop and found only very small differences to cutoff regularization.) The contributions of eqs.(13, 14) to the nucleon electric and magnetic form factor can then be identified after making use of the Dirac equation, eq.(7), to simplify the numerators of the Feynman integrals.

Before discussing results for the baryon charge radii and magnetic moments, it is instructive to consider the dependence of these quantities on the baryon c.o.m. momentum. Fig.1 shows, for a fixed quark and diquark mass, the proton charge radius from eq.(12) as a function of $P^2 \equiv P_i^2 = P_f^2 \neq m_B^2$. Here, the amplitudes are normalized according to eq.(9), but at $P^2 \neq m_B^2$. (One may imagine the off–shell values of P^2 to be generated by varying the strength of the static quark exchange term, M, in eq.(7).) The charge radius grows and eventually diverges, as P^2 approaches the diquark–quark continuum threshold. The presence of a singularity is, of course, due to the lack of confinement of this model. The strong variation of $\langle r^2 \rangle_p$ with P^2 for bound baryons, however, should not be regarded as unphysical. We know that even in confining theories there are "would–be"–thresholds related to losely bound states [13]. In this spirit, fig.1 simply reflects the fact that, in the context of the effective quark dynamics described by the NJL model, the baryon has to be considered as a losely bound state, contrary to *e.g.* the pion. This is in agreement with the fact that the diquark masses obtained in the NJL model [14] as well as in the instanton vacuum [15] are typically of the order of $0.5 \ldots 0.6 \, \mathrm{GeV}$, so that the diquark–quark threshold is close to the physical nucleon mass. Furthermore, the exact Faddeev calculation leads to nucleon binding energies of $\varepsilon_B = M + m_{0^+} - m_B \approx 50 \ldots 100 \, \mathrm{MeV} \ll m_B$ [8]. When fixing the parameter Λ_B of the static approximation, we must keep in mind that it is the binding energy — not the absolute mass — which is smoothly related to

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[2]Note that the off–shell dependence of the exact $\mathcal{D}(k)$ and $\mathcal{F}^\mu(k, k')$ cancels to leading order in $(k^2 - m_d^2)$ and $(k'^2 - m_d^2)$, when considering the product $\mathcal{D}(k)\mathcal{F}^\mu(k, k')\mathcal{D}(k')$ [12].

the characteristics of the baryon wave function. We thus take values of Λ_B, for which the Dirac equation, eq.(7) gives binding energies comparable to those of the exact solutions (see fig.1). These lead to realistic charge radii. We see no physical grounds for identifying Λ_B with the NJL cutoff employed in the diquark and meson sector [7], the more since values of $\Lambda_B \approx 0.6 \ldots 0.7\,\text{GeV}$ would considerably overestimate the binding energy, *cf.* fig.1 and [8]. For a given value of Λ_B, we then choose the NJL coupling constant in the diquark channel such as to reproduce the physical nucleon mass in eq.(7) [7]. We emphasize that, in this way, the baryon form factors are not sensitive to the precise value of the baryon mass, if Λ_B (and thus, approximately, ε_B) is kept fixed.

With the parameters of the model determined in this way, we may now discuss the results for the on–shell nucleon charge radii and magnetic moments. As expected, the proton charge radius depends inversely on the cutoff used in the diquark–quark wave function. The experimental radius, $\langle r^2 \rangle_p = 0.74\,\text{fm}^2$, can be reproduced for $\Lambda_B \approx 0.32\,\text{GeV}$. Since this value for Λ_B would probably be larger, if the 1^+–diquarks were included [11], we give in table 1 also results for larger values of Λ_B. In addition, we have performed calculations with a pointlike diquark form factor, $F_{0^+} \equiv 1$, *cf.* eq.(15). Comparison of the values obtained with extended and pointlike scalar diquarks shows that the intrinsic charge radius of the scalar diquark contributes about $10 \ldots 20\%$ of the proton charge radius. The neutron charge radius is reproduced well. This quantity originally provided evidence for scalar diquark correlations [16]. The intrinsic diquark radius somewhat increases the neutron charge radius, improving the agreement with the experimental value. The magnetic moments are not too different from their values in an additive diquark–quark picture with only 0^+–diquarks, $\mu_p = \frac{2}{3}(e/2M), \mu_n = -\frac{1}{3}(e/2M)$. Here, large contributions are to be expected from the 1^+–diquark channel. Contrary to the charge radii, the magnetic moments are rather independent of the size of the baryon wave function.

We have also calculated the isovector axial coupling constant of the nucleon[3], g_A. In this case the quark vertex is given by $\Gamma_q^{5\mu} = i\tau^a \gamma_5 \gamma^\mu$. The scalar diquark has no axial coupling, so that the baryon axial form factor is determined entirely by the corresponding valence quark axial current, eq.(14). (We do not include a counter term in eq.(12) now.) As shown in table 1, for realistic parameters, $g_A < 1$. The value $g_A = 1$ would be obtained in an additive diquark–quark scheme with only 0^+–diquarks. Corrections to this value are seen to increase with Λ_B, as the diquark–quark system becomes more strongly bound. Note that also in g_A we expect sizable contributions from the 1^+–diquark channel, both through the axial coupling of the 1^+–diquark as well as through 0^+–1^+ transitions.

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[3]We do not address here questions related to the induced pseudoscalar form factor and the Goldberger–Treiman relation, which require the inclusion of meson fluctuations, but stay within a valence quark picture. See [17] for a discussion of these issues.

In summary, our results show two types of behavior of nucleon observables. Charge radii depend strongly on the extent of the diquark–quark wave function and can reasonably be described in a picture with (even pointlike) 0^+–diquarks. On the contrary, magnetic moments and g_A are rather insensitive to the strength of the diquark–quark binding and therefore amenable to an additive diquark–quark description [11]. They require, however, the inclusion of the 1^+–diquarks.

In our simple estimates, we have treated the cutoff, Λ_B, as a parameter and related it to the proton charge radius. In principle, the width of the diquark–quark wave function should be determined by the underlying effective quark dynamics. It remains to be seen whether the quark exchange mechanism included in eq.(4) leads by itself to realistic baryon radii, or whether these quantities are dominated by other effects (dynamical meson exchange, confinement). Meanwhile, it is of practical interest that, with appropriate values of Λ_B, the static approximation seems to offer the possibility of a reasonable approximation to the nucleon wave function, if 1^+–diquarks are included. It may *e.g.* be employed to calculate strong form factors of the nucleon.

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	$\Lambda_B = 0.323\,\mathrm{GeV}$	$\Lambda_B = 0.35\,\mathrm{GeV}$	$\Lambda_B = 0.45\,\mathrm{GeV}$	exp.
g_2/g_1	1.46	1.42	1.27	
m_{0^+}/GeV	0.582	0.593	0.631	
$Z_{0^+}^{1/2}$	14.3	14.0	13.0	
$\varepsilon_B/\mathrm{GeV}$	0.044	0.055	0.093	
$\langle r^2 \rangle_p/\mathrm{fm}^2$	0.74 (0.66)	0.65 (0.57)	0.48 (0.40)	0.74
$\langle r^2 \rangle_n/\mathrm{fm}^2$	$-0.11\,(-0.19)$	$-0.10\,(-0.17)$	$-0.07\,(-0.15)$	-0.12
$\mu_p/(e/2m_B)$	1.67	1.67	1.75	2.79
$\mu_n/(e/2m_B)$	-0.78	-0.82	-0.83	-1.91
g_A	0.96	0.95	0.91	1.26

Table 1: *The nucleon electric charge radii, $\langle r^2 \rangle_{p,n}$, magnetic moments, $\mu_{p,n}$, and isovector axial coupling constant, g_A, for various values of the cutoff in the diquark–quark momentum, Λ_B. Results are given for a constituent quark mass of $M = 0.4\,\mathrm{GeV}$. For each value of Λ_B, the coupling constant of the scalar diquark channel, g_2, has been chosen to fit the physical nucleon mass, $m_B = m_p = 0.938\,\mathrm{GeV}$. The corresponding values of m_{0^+}, Z_{0^+}, and the diquark form factor have been determined from the full diquark propagator of the proper–time regularized NJL model [11]. The ratio of the diquark coupling constant to that in the meson channel, g_2/g_1, is as defined in [7, 11]. Also given is the baryon binding energy, $\varepsilon_B = M + m_{0^+} - m_p$. With a value of $\Lambda_B = 0.323\,\mathrm{GeV}$, the experimental proton charge radius can be reproduced. The charge radii in parentheses have been calculated with a pointlike diquark form factor, $F_{0^+} \equiv 1$, cf. eq.(15).*

Figure caption

Fig. 1: *The dependence of the proton charge radius, $\langle r^2 \rangle_p$ on the proton c.o.m. momentum, $P^2 \equiv P_{\rm f}^2 = P_{\rm i}^2 \neq m_B^2$. The charge radius is calculated from* eq.(12) *for arbitrary P^2, with amplitudes normalized according to* eq.(9)*, but at $P^2 \neq m_B^2$. Shown are the radii obtained with $\Lambda_B = 0.55\,{\rm GeV}$ (solid line), $0.45\,{\rm GeV}$ (dashed line) and $0.35\,{\rm GeV}$ (dotted line). The on–shell masses corresponding to the different values of Λ_B are marked by squares on the respective curves. Here, for all Λ_B, the constituent quark mass is $M = 0.4\,{\rm GeV}$, and for the diquark mass we have chosen a fixed value of $m_d = 0.6\,{\rm GeV}$, so that the diquark–quark continuum starts at $1.0\,{\rm GeV}$.*
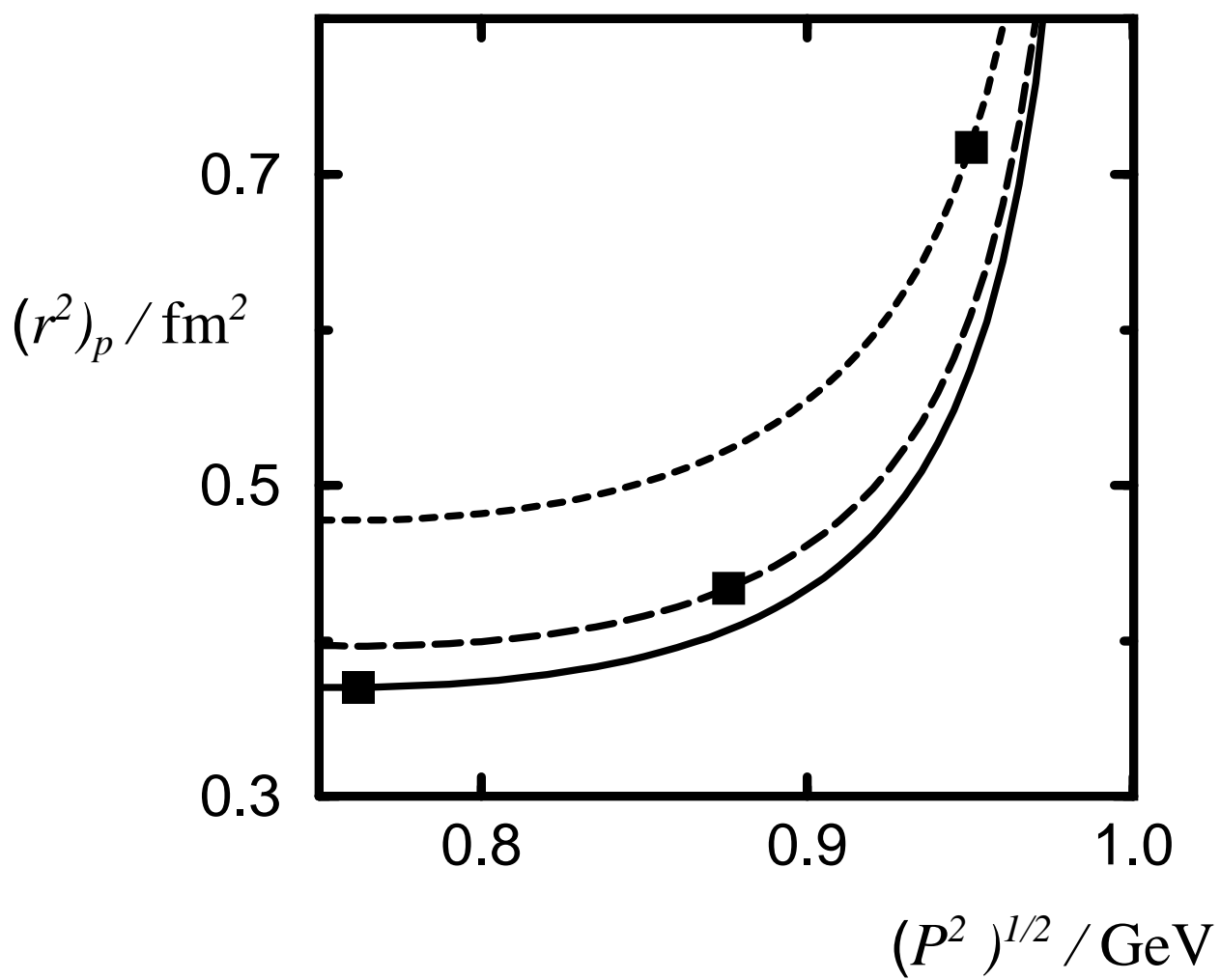
This figure "fig1-1.png" is available in "png" format from:

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Fig. 1